3

Motion in a Plane

TOPIC 1 Vectors

1. A force $\vec{F} = (\hat{i} + 2\hat{j} + 3\hat{k})$ N acts at a point $(4\hat{i} + 3\hat{j} - \hat{k})$ m. Then the magnitude of torque about the point $(\hat{i} + 2\hat{j} + \hat{k})$ m will be $\sqrt{x}$ N-m. The value of x is ______.

[NA Sep. 05, 2020 (I)]

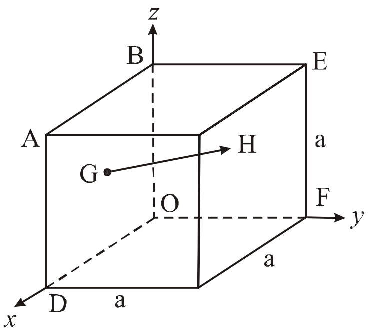
2. The sum of two forces $\vec{P}$ and $\vec{Q}$ is $\vec{R}$ such that $|\vec{R}| = |\vec{P}|$. The angle θ (in degrees) that the resultant of 2 $\vec{P}$ and $\vec{Q}$ will make with $\vec{Q}$ is _______.

[NA 7 Jan. 2020 II]

3. Let $|\vec{A_1}| = 3$, $|\vec{A_2}| = 5$ and $|\vec{A_1} + \vec{A_2}| = 5$. The value of $(2\vec{A_1} + 3\vec{A_2}) \bullet (3\vec{A_1} - 2\vec{A_2})$ is : **[8 April 2020 II]**

(a) -106.5 (b) -99.5
(c) -112.5 (d) -118.5

4. In the cube of side 'a' shown in the figure, the vector from the central point of the face ABOD to the central point of the face BEFO will be: **[10 Jan. 2019 I]**



(a) $\frac{1}{2}a(\hat{k} - \hat{i})$ (b) $\frac{1}{2}a(\hat{i} - \hat{k})$
(c) $\frac{1}{2}a(\hat{j} - \hat{i})$ (d) $\frac{1}{2}a(\hat{j} - \hat{k})$

5. Two forces P and Q, of magnitude 2F and 3F, respectively, are at an angle θ with each other. If the force Q is doubled, then their resultant also gets doubled. Then, the angle θ is: **[10 Jan. 2019 II]**

(a) 120° (b) 60°
(c) 90° (d) 30°

6. Two vectors $\vec{A}$ and $\vec{B}$ have equal magnitudes. The magnitude of $(\vec{A} + \vec{B})$ is 'n' times the magnitude of $(\vec{A} - \vec{B})$. The angle between $\vec{A}$ and $\vec{B}$ is: **[10 Jan. 2019 II]**

(a) $\cos^{-1}\left[\frac{n^2-1}{n^2+1}\right]$ (b) $\cos^{-1}\left[\frac{n-1}{n+1}\right]$
(c) $\sin^{-1}\left[\frac{n^2-1}{n^2+1}\right]$ (d) $\sin^{-1}\left[\frac{n-1}{n+1}\right]$

7. Let $\vec{A} = (\hat{i} + \hat{j})$ and $\vec{B} = (\hat{i} - \hat{j})$. The magnitude of a coplanar vector $\vec{C}$ such that $\vec{A}.\vec{C} = \vec{B}.\vec{C} = \vec{A}.\vec{B}$ is given by **[Online April 16, 2018]**

(a) $\sqrt{\frac{5}{9}}$ (b) $\sqrt{\frac{10}{9}}$
(c) $\sqrt{\frac{20}{9}}$ (d) $\sqrt{\frac{9}{12}}$

8. A vector $\vec{A}$ is rotated by a small angle Δθ radian (Δθ << 1) to get a new vector $\vec{B}$. In that case $|\vec{B} - \vec{A}|$ is : **[Online April 11, 2015]**

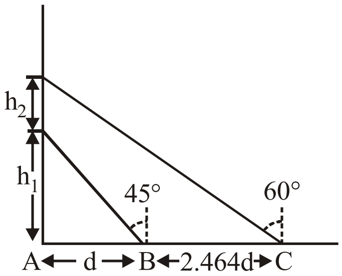
(a) $|\vec{A}|\Delta\theta$ (b) $|\vec{B}|\Delta\theta - |\vec{A}|$
(c) $|\vec{A}|\left(1 - \frac{\Delta\theta^2}{2}\right)$ (d) 0

9. If $\vec{A} \times \vec{B} = \vec{B} \times \vec{A}$, then the angle between A and B is **[2004]**

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$
(c) π (d) $\frac{\pi}{4}$

TOPIC 2 Motion in a Plane with Constant Acceleration

10. A balloon is moving up in air vertically above a point A on the ground. When it is at a height h_1, a girl standing at a distance d (point B) from A (see figure) sees it at an angle 45º with respect to the vertical. When the balloon climbs up a further height h_2, it is seen at an angle 60º with respect to the vertical if the girl moves further by a distance 2.464 d (point C). Then the height h_2 is (given tan 30º = 0.5774): **[Sep. 05, 2020 (I)]**



(a) 1.464 d (b) 0.732 d
(c) 0.464 d (d) d

11. Starting from the origin at time $t = 0$, with initial velocity $5\hat{j}$ ms^{-1}, a particle moves in the x–y plane with a constant acceleration of $(10\hat{i}+4\hat{j})$ ms^{-2}. At time t, its coordiantes are (20 m, y_0 m). The values of t and y_0 are, respectively : **[Sep. 04, 2020 (I)]**

(a) 2 s and 18 m (b) 4 s and 52 m
(c) 2 s and 24 m (d) 5 s and 25 m

12. The position vector of a particle changes with time according to the relation $\vec{r}(t) = 15t^2\hat{i} + (4-20t^2)\hat{j}$. What is the magnitude of the acceleration at $t = 1$? **[9 April 2019 II]**

(a) 40 (b) 25 (c) 100 (d) 50

13. A particle moves from the point $(2.0\hat{i}+4.0\hat{j})$m, at t = 0, with an initial velocity $(5.0\hat{i}+4.0\hat{j})$ms^{-1}. It is acted upon by a constant force which produces a constant acceleration $(4.0\hat{i}+4.0\hat{j})$ms^{-2}. What is the distance of the particle from the origin at time 2s? **[11 Jan. 2019 II]**

(a) 15m (b) $20\sqrt{2}$m
(c) 5m (d) $10\sqrt{2}$m

14. A particle is moving with a velocity $\vec{v} = K(y\hat{i} + x\hat{j})$, where K is a constant. The general equation for its path is: **[9 Jan. 2019 I]**

(a) $y = x^2$ + constant (b) $y^2 = x$ + constant
(c) $y^2 = x^2$ + constant (d) xy = constant

15. A particle starts from the origin at $t = 0$ with an initial velocity of $3.0\hat{i}$ m/s and moves in the x-y plane with a constant acceleration $(6.0\hat{i}+4.0\hat{j})$ m/s². The x-coordinate of the particle at the instant when its y-coordinate is 32 m is D meters. The value of D is: **[9 Jan. 2020 II]**

(a) 32 (b) 50 (c) 60 (d) 40

16. A particle is moving along the x-axis with its coordinate with time 't' given by $x(t) = 10 + 8t - 3t^2$. Another particle is moving along the y-axis with its coordinate as a function of time given by $y(t) = 5 - 8t^3$. At $t = 1$ s, the speed of the second particle as measured in the frame of the first particle is given as $\sqrt{v}$. Then v (in m/s) is____ **[NA 8 Jan. 2020 I]**

17. A particle moves such that its position vector $\vec{r}(t) = \cos\omega t\,\hat{i} + \sin\omega t\,\hat{j}$ where ω is a constant and t is time. Then which of the following statements is true for the velocity $\vec{v}(t)$ and acceleration $\vec{a}(t)$ of the particle: **[8 Jan. 2020 II]**

(a) $\vec{v}$ is perpendicular to $\vec{r}$ and $\vec{a}$ is directed away from the origin
(b) $\vec{v}$ and $\vec{a}$ both are perpendicular to $\vec{r}$
(c) $\vec{v}$ and $\vec{a}$ both are parallel to $\vec{r}$
(d) $\vec{v}$ is perpendicular to $\vec{r}$ and $\vec{a}$ is directed towards the origin

18. A particle is moving with velocity $\vec{v} = k(y\hat{i} + x\hat{j})$, where k is a constant. The general equation for its path is **[2010]**

(a) $y = x^2$ + constant (b) $y^2 = x$ + constant
(c) xy = constant (d) $y^2 = x^2$ + constant

19. A particle has an initial velocity of $3\hat{i}+4\hat{j}$ and an acceleration of $0.4\hat{i}+0.3\hat{j}$. Its speed after 10 s is : **[2009]**

(a) $7\sqrt{2}$ units (b) 7 units
(c) 8.5 units (d) 10 units

20. The co-ordinates of a moving particle at any time 't' are given by $x = \alpha t^3$ and $y = \beta t^3$. The speed of the particle at time 't' is given by **[2003]**

(a) $3t\sqrt{\alpha^2+\beta^2}$ (b) $3t^2\sqrt{\alpha^2+\beta^2}$
(c) $t^2\sqrt{\alpha^2+\beta^2}$ (d) $\sqrt{\alpha^2+\beta^2}$

TOPIC 3 Projectile Motion

21. A particle of mass m is projected with a speed u from the ground at an angle $\theta = \frac{\pi}{3}$ w.r.t. horizontal (x-axis). When it has reached its maximum height, it collides completely inelastically with another particle of the same mass and velocity $u\hat{i}$. The horizontal distance covered by the combined mass before reaching the ground is: **[9 Jan. 2020 II]**

(a) $\frac{3\sqrt{3}}{8}\frac{u^2}{g}$ (b) $\frac{3\sqrt{2}}{4}\frac{u^2}{g}$

(c) $\frac{5}{8}\frac{u^2}{g}$ (d) $2\sqrt{2}\frac{u^2}{g}$

22. The trajectory of a projectile near the surface of the earth is given as $y = 2x - 9x^2$. If it were launched at an angle θ_0 with speed v_0 then ($g = 10\ ms^{-2}$): **[12 April 2019 I]**

(a) $\theta_0 = \sin^{-1}\frac{1}{\sqrt{5}}$ and $v_0 = \frac{5}{3}\ ms^{-1}$

(b) $\theta_0 = \cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and $v_0 = \frac{3}{5}\ ms^{-1}$

(c) $\theta_0 = \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$ and $v_0 = \frac{9}{3}\ ms^{-1}$

(d) $\theta_0 = \sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and $v_0 = \frac{3}{5}\ ms^{-1}$

23. A shell is fired from a fixed artillery gun with an initial speed u such that it hits the target on the ground at a distance R from it. If t_1 and t_2 are the values of the time taken by it to hit the target in two possible ways, the product t_1t_2 is : **[12 April 2019 I]**

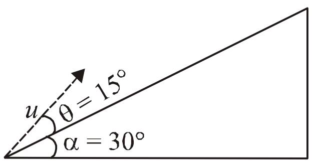
(a) R/4g (b) R/g (c) R/2g (d) 2R/g

24. Two particles are projected from the same point with the same speed u such that they have the same range R, but different maximum heights, h_1 and h_2. Which of the following is correct ? **[12 April 2019 II]**

(a) $R^2 = 4\,h_1h_2$ (b) $R^2 = 16\,h_1h_2$

(c) $R^2 = 2\,h_1h_2$ (d) $R^2 = h_1h_2$

25. A plane is inclined at an angle $\alpha = 30^o$ with respect to the horizontal. A particle is projected with a speed $u = 2\ ms^{-1}$, from the base of the plane, as shown in figure. The distance from the base, at which the particle hits the plane is close to : (Take $g = 10\ ms^{-2}$) **[10 April 2019 II]**



(a) 20 cm (b) 18 cm (c) 26 cm (d) 14 cm

26. A body is projected at t = 0 with a velocity $10\ ms^{-1}$ at an angle of 60° with the horizontal. The radius of curvature of its trajectory at t = 1s is R. Neglecting air resistance and taking acceleration due to gravity $g = 10\ ms^{-2}$, the value of R is: **[11 Jan. 2019 I]**

(a) 10.3 m (b) 2.8 m

(c) 2.5 m (d) 5.1 m

27. Two guns A and B can fire bullets at speeds 1 km/s and 2 km/s respectively. From a point on a horizontal ground, they are fired in all possible directions. The ratio of maximum areas covered by the bullets fired by the two guns, on the ground is: **[10 Jan. 2019 I]**

(a) 1 : 16 (b) 1 : 2 (c) 1 : 4 (d) 1 : 8

28. The initial speed of a bullet fired from a rifle is 630 m/s. The rifle is fired at the centre of a target 700 m away at the same level as the target. How far above the centre of the target ? **[Online April 11, 2014]**

(a) 1.0 m (b) 4.2 m (c) 6.1 m (d) 9.8 m

29. The position of a projectile launched from the origin at $t = 0$ is given by $\vec{r} = \left(40\hat{i} + 50\hat{j}\right)$ m at $t = 2$s. If the projectile was launched at an angle θ from the horizontal, then θ is (take $g = 10\ ms^{-2}$) **[Online April 9, 2014]**

(a) $\tan^{-1}\frac{2}{3}$ (b) $\tan^{-1}\frac{3}{2}$

(c) $\tan^{-1}\frac{7}{4}$ (d) $\tan^{-1}\frac{4}{5}$

30. A projectile is given an initial velocity of $(\hat{i} + 2\hat{j})$ m/s, where $\hat{i}$ is along the ground and $\hat{j}$ is along the vertical. If $g = 10\ m/s^2$, the equation of its trajectory is : **[2013]**

(a) $y = x - 5x^2$ (b) $y = 2x - 5x^2$

(c) $4y = 2x - 5x^2$ (d) $4y = 2x - 25x^2$

31. The maximum range of a bullet fired from a toy pistol mounted on a car at rest is $R_0 = 40$ m. What will be the acute angle of inclination of the pistol for maximum range when the car is moving in the direction of firing with uniform velocity v = 20 m/s, on a horizontal surface ? ($g = 10\ m/s^2$) **[Online April 25, 2013]**

(a) 30° (b) 60° (c) 75° (d) 45°

32. A ball projected from ground at an angle of 45° just clears a wall in front. If point of projection is 4 m from the foot of wall and ball strikes the ground at a distance of 6 m on the other side of the wall, the height of the wall is : **[Online April 22, 2013]**

(a) 4.4 m (b) 2.4 m (c) 3.6 m (d) 1.6 m

33. A boy can throw a stone up to a maximum height of 10 m. The maximum horizontal distance that the boy can throw the same stone up to will be **[2012]**

(a) $20\sqrt{2}$ m (b) 10 m

(c) $10\sqrt{2}$ m (d) 20 m

34. A water fountain on the ground sprinkles water all around it. If the speed of water coming out of the fountain is v, the total area around the fountain that gets wet is: **[2011]**

(a) $\pi\frac{v^4}{g^2}$ (b) $\frac{\pi}{2}\frac{v^4}{g^2}$ (c) $\pi\frac{v^2}{g^2}$ (d) $\pi\frac{v^2}{g}$

35. A projectile can have the same range 'R' for two angles of projection. If 'T_1' and 'T_2' to be time of flights in the two cases, then the product of the two time of flights is directly proportional to. **[2004]**

(a) R (b) $\frac{1}{R}$ (c) $\frac{1}{R^2}$ (d) R^2

36. A ball is thrown from a point with a speed 'v_0' at an elevation angle of θ. From the same point and at the same instant, a person starts running with a constant speed $\frac{'v_0'}{2}$ to catch the ball. Will the person be able to catch the ball? If yes, what should be the angle of projection θ? **[2004]**

(a) No (b) Yes, 30°
(c) Yes, 60° (d) Yes, 45°

37. A boy playing on the roof of a 10 m high building throws a ball with a speed of 10m/s at an angle of 30º with the horizontal. How far from the throwing point will the ball be at the height of 10 m from the ground ? **[2003]**

$[g = 10\text{m/s}^2, \sin 30^o = \frac{1}{2}, \cos 30^o = \frac{\sqrt{3}}{2}]$

(a) 5.20m (b) 4.33m (c) 2.60m (d) 8.66m

TOPIC 4 Relative Velocity in Two Dimensions & Uniform Circular Motion

38. A clock has a continuously moving second's hand of 0.1 m length. The average acceleration of the tip of the hand (in units of ms^{-2}) is of the order of: **[Sep. 06, 2020 (I)]**

(a) 10^{-3} (b) 10^{-4}
(c) 10^{-2} (d) 10^{-1}

39. When a carsit at rest, its driver sees raindrops falling on it vertically. When driving the car with speed v, he sees that raindrops are coming at an angle 60º from the horizontal. On furter increasing the speed of the car to $(1 + \beta)v$, this angle changes to 45º. The value of β is close to: **[Sep. 06, 2020 (II)]**

(a) 0.50 (b) 0.41
(c) 0.37 (d) 0.73

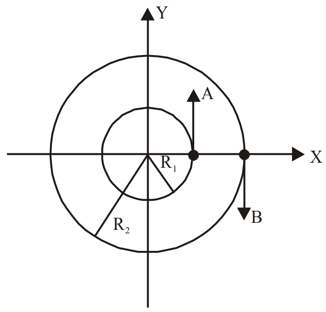
40. The stream of a river is flowing with a speed of 2 km/h. A swimmer can swim at a speed of 4 km/h. What should be the direction of the swimmer with respect to the flow of the river to cross the river straight? **[9 April 2019 I]**

(a) 90° (b) 150°
(c) 120° (d) 60°

41. Ship A is sailing towards north-east with velocity km/hr where points east and , north. Ship B is at a distance of 80 km east and 150 km north of Ship A and is sailing towards west at 10 km/hr. A will be at minimum distance from B in: **[8 April 2019 I]**

(a) 4.2 hrs. (b) 2.6 hrs.
(c) 3.2 hrs. (d) 2.2 hrs.

42. Two particles A, B are moving on two concentric circles of radii R_1 and R_2 with equal angular speed ω. At t = 0, their positions and direction of motion are shown in the figure : **[12 Jan. 2019 II]**



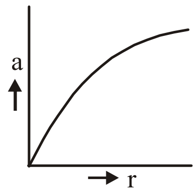
The relative velocity $\vec{v_A} - \vec{v_B}$ and $t = \frac{\pi}{2\omega}$ is given by:

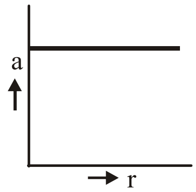
(a) $\omega(R_1 + R_2)\hat{i}$ (b) $-\omega(R_1 + R_2)\hat{i}$
(c) $\omega(R_2 - R_1)\hat{i}$ (d) $\omega(R_1 - R_2)\hat{i}$

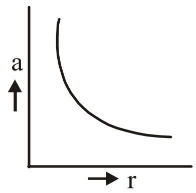
43. A particle is moving along a circular path with a constant speed of 10 ms^{-1}. What is the magnitude of the change in velocity of the particle, when it moves through an angle of 60° around the centre of the circle? **[Online April 10, 2015]**

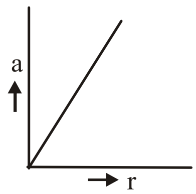
(a) $10\sqrt{3}$m/s (b) zero
(c) $10\sqrt{2}$m/s (d) 10 m/s

44. If a body moving in circular path maintains constant speed of 10 ms^{-1}, then which of the following correctly describes relation between acceleration and radius? **[Online April 10, 2015]**

(a) 

(b) 

(c) 

(d) 

Hints & Solutions

1. **(195)**

Given : $\vec{F} = (\hat{i} + 2\hat{j} + 3\hat{k})$ N

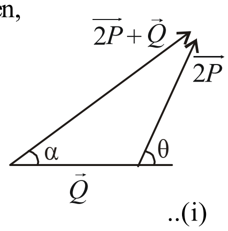
And, $\vec{r} = [(4\hat{i} + 3\hat{j} - \hat{k}) - (\hat{i} + 2\hat{j} + \hat{k})] = 3\hat{i} + \hat{j} - 2\hat{k}$

Torque, $\tau = \vec{r} \times \vec{F} = (3\hat{i} + \hat{j} - 2\hat{k}) \times (\hat{i} + 2\hat{j} + 3\hat{k})$

$$\tau = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 1 & 2 & 3 \end{vmatrix} = 7\hat{i} - 11\hat{j} + 5\hat{k}$$

Magnitude of torque, $|\vec{\tau}| = \sqrt{195}$.

2. **(90)** Given,



$|\vec{R}| = |\vec{P}| \Rightarrow |\vec{P} + \vec{Q}| = |\vec{P}|$

$P^2 + Q^2 + 2PQ.\cos\theta = P^2$

$\Rightarrow Q + 2P\cos\theta = 0$

$\Rightarrow \cos\theta = -\dfrac{Q}{2P}$ ..(i)

$\tan\alpha = \dfrac{2P\sin\theta}{Q + 2P\cos\theta} = \infty$ ($\because 2P\cos\theta + Q = 0$)

$\Rightarrow \alpha = 90°$

3. **(d)** Using,

$R^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\theta$

$5^2 = 3^2 + 5^2 + 2 \times 3 \times 5\cos\theta$

or $\cos\theta = -0.3$

$\left(2\vec{A_1} + 3\vec{A_2}\right) \cdot \left(3\vec{A_1} - 2\vec{A_2}\right) = 2A_1 \times 3A_1$

$+ (3A_2)(3A_1)\cos\theta - (2A_1)(2A_2)\cos\theta - 3A_2 \times 2A_2$

$= 6A_1^2 + 9A_1A_2\cos\theta - 4A_1A_2\cos\theta - 6A_2^2$

$= 6A_1^2\ 6A_2^2 + 5A_1A_2\cos\theta$

$= 6 \times 3^2 - 6 \times 5^2 + 5 \times 3 \times 5(-0.3)$

$= -118.5$

4. **(c)** From figure,

$\vec{r}_G = \dfrac{a}{2}\hat{i} + \dfrac{a}{2}\hat{k}$

$\vec{r}_H = \dfrac{a}{2}\hat{j} + \dfrac{a}{2}\hat{k}$

$\therefore \vec{r}_H - \vec{r}_G = \left(\dfrac{a}{2}\hat{j} + \dfrac{a}{2}\hat{k}\right) - \left(\dfrac{a}{2}\hat{i} + \dfrac{a}{2}\hat{k}\right) = \dfrac{a}{2}(\hat{j} - \hat{i})$

5. **(a)** Using, $R^2 = P^2 + Q^2 + 2PQ\cos\theta$

$4F^2 + 9F^2 + 12F^2\cos\theta = R^2$

When forces Q is doubled,

$4F^2 + 36F^2 + 24F^2\cos\theta = 4R^2$

$4F^2 + 36F^2 + 24F^2\cos\theta$

$= 4(13F^2 + 12F^2\cos\theta) = 52F^2 + 48F^2\cos\theta$

$\therefore \cos\theta = -\dfrac{12F^2}{24F^2} = -\dfrac{1}{2} \Rightarrow \theta = 120^o$

6. **(a)** Let magnitude of two vectors $\vec{A}$ and $\vec{B}$ = a

$|\vec{A} + \vec{B}| = \sqrt{a^2 + a^2 + 2a^2\cos\theta}$ and

$|\vec{A} - \vec{B}| = \sqrt{a^2 + a^2 - 2a^2[\cos(180° - \theta)]}$

$= \sqrt{a^2 + a^2 - 2a^2\cos\theta}$

and accroding to question,

$|\vec{A} + \vec{B}| = n|\vec{A} - \vec{B}|$

or, $\dfrac{a^2 + a^2 + 2a^2\cos\theta}{a^2 + a^2 - 2a^2\cos\theta} = n^2$

$\Rightarrow \dfrac{a^2(1 + 1 + 2\cos\theta)}{a^2(1 + 1 - 2\cos\theta)} n^2 \Rightarrow \dfrac{(1 + \cos\theta)}{(1 - \cos\theta)} = n^2$

using componendo and dividendo theorem, we get

$\theta = \cos^{-1}\left(\dfrac{n^2 - 1}{n^2 + 1}\right)$

7. **(a)** If $\vec{C} = a\hat{i} + b\hat{j}$ then $\vec{A}.\vec{C} = \vec{A}.\vec{B}$

$a + b = 1$ ..... (i)

$\vec{B}.\vec{C} = \vec{A}.\vec{B}$

$2a - b = 1$ ..... (ii)

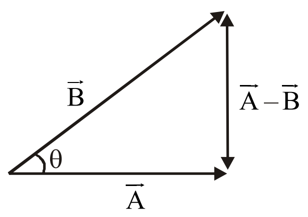
Solving equation (i) and (ii) we get

$a = \dfrac{1}{3}, b = \dfrac{2}{3}$

$\therefore$ Magnitude of coplanar vector, $|\vec{C}| = \sqrt{\dfrac{1}{9} + \dfrac{4}{9}} = \sqrt{\dfrac{5}{9}}$

8. (a) Arc length = radius × angle

So, $|\vec{B} - \vec{A}| = |\vec{A}|\Delta\theta$



9. **(c)** $\vec{A} \times \vec{B} - \vec{B} \times \vec{A} = 0 \Rightarrow \vec{A} \times \vec{B} + \vec{A} \times \vec{B} = 0$

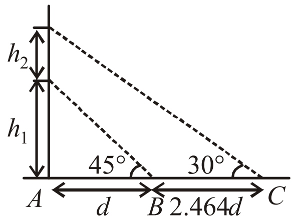
$\therefore \vec{A} \times \vec{B} = 0$

Angle between them is 0, π, or 2 π

from the given options, $\theta = \pi$

10. **(d)** From figure/ trigonometry,

$\frac{h_1}{d} = \tan 45°$ $\therefore h_1 = d$



And, $\frac{h_1 + h_2}{d + 2.464d} = \tan 30°$

$\Rightarrow (h_1 + h_2) \times \sqrt{3} = 3.46d$

$\Rightarrow (h_1 + h_2) = \frac{3.46d}{\sqrt{3}} \Rightarrow d + h_2 = \frac{3.46d}{\sqrt{3}}$

$\therefore h_2 = d$

11. **(a)** Given : $\vec{u} = 5\hat{j}$ m/s

Acceleration, $\vec{a} = 10\hat{i} + 4\hat{j}$ and

final coordinate $(20, y_0)$ in time t.

$S_x = u_x t + \frac{1}{2} a_x t^2$ $[\because u_x = 0]$

$\Rightarrow 20 = 0 + \frac{1}{2} \times 10 \times t^2 \Rightarrow t = 2$ s

$S_y = u_y \times t + \frac{1}{2} a_y t^2$

$y_0 = 5 \times 2 + \frac{1}{2} \times 4 \times 2^2 = 18$ m

12. **(d)** $\vec{r} = 15t^2\hat{i} + (4 - 20t^2)\hat{j}$

$\vec{v} = \frac{d\vec{r}}{dt} = 30t\hat{i} - 40t\hat{j}$

Acceleration, $\vec{a} = \frac{d\vec{v}}{dt} = 30\hat{i} - 40\hat{j}$

$\therefore a = \sqrt{30^2 + 40^2} = 50 \text{ m/s}^2$

13. **(b)** As $\vec{S} = \vec{u}t + \frac{1}{2}\vec{a}t^2$

$\vec{S} = (5\hat{i} + 4\hat{j})2 + \frac{1}{2}(4\hat{i} + 4\hat{j})4$

$= 10\hat{i} + 8\hat{j} + 8\hat{i} + 8\hat{j}$

$\vec{r}_f - \vec{r}_i = 18\hat{i} + 16\hat{j}$

[as $\vec{s}$ = change in position $= \vec{r}_f - \vec{r}_i$]

$\vec{r}_f = 20\hat{i} + 20\hat{j}$

$|\vec{r}_f| = 20\sqrt{2}$

14. **(c)** From given equation,

$\vec{V} = K\left(y\hat{i} + x\hat{j}\right)$

$\frac{dx}{dt} = ky$ and $\frac{dy}{dt} = kx$

Now $\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{x}{y} = \frac{dy}{dx}, \Rightarrow ydy = xdx$

Integrating both side

$y^2 = x^2 + c$

15. **(c)** Using $S = ut + \frac{1}{2}at^2$

$y = u_y t + \frac{1}{2} a_y t^2$ (along y Axis)

$\Rightarrow 32 = 0 \times t + \frac{1}{2}(4)t^2$

$\Rightarrow \frac{1}{2} \times 4 \times t^2 = 32$

$\Rightarrow t = 4$ s

$S_x = u_x t + \frac{1}{2} a_x t^2$ (Along x Axis)

$\Rightarrow x = 3 \times 4 + \frac{1}{2} \times 6 \times 4^2 = 60$

16. **(580)**

For pariticle 'A'	For particle 'B'
$X_A = -3t^2 + 8t + 10$	$Y_B = 5 - 8t^3$
$\vec{V}_A = (8 - 6t)\hat{i}$	$\vec{V}_B = -24t^2\hat{j}$
$\vec{a}_A = -6\hat{i}$	$\vec{a}_B = -48t\hat{j}$

At $t = 1$ sec

$\vec{V}_A = (8 - 6t)\hat{i} = 2\hat{i}$ and $\vec{v}_B = -24\hat{j}$

$\therefore \vec{V}_{B/A} = -\vec{v}_A + \vec{v}_B = -2\hat{i} - 24\hat{j}$

$\therefore$ Speed of B w.r.t. A, $\sqrt{v} = \sqrt{2^2 + 24^2}$

$= \sqrt{4 + 576} = \sqrt{580}$

$\therefore v = 580$ (m/s)

17. **(d)** Given, Position vector,

$\vec{r} = \cos\omega t\hat{i} + \sin\omega t\,\hat{j}$

Velocity, $\vec{v} = \frac{d\vec{r}}{dt} = \omega(-\sin\omega t\hat{i} + \cos\omega t\,\hat{j})$

Acceleration,

$\vec{a} = \frac{d\vec{v}}{dt} = -\omega^2(\cos\omega t\hat{i} + \sin\omega t\,\hat{j})$

$\vec{a} = -\omega^2\vec{r}$

$\therefore$ $\vec{a}$ is antiparallel to $\vec{r}$

Also $\vec{v}.\vec{r} = 0$ $\therefore \vec{v} \perp \vec{r}$

Thus, the particle is performing uniform circular motion.

18. (d) $v = k(yi + xj)$

$v = kyi + kxj$

$$\frac{dx}{dt} = ky, \frac{dy}{dt} = kx$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$\frac{dy}{dx} = \frac{kx}{ky}$$

$ydy = xdx$ ...(*i*)

Integrating equation (i)

$$\int ydy = \int x \cdot dx$$

$$y^2 = x^2 + c$$

19. (a) Given $\vec{u} = 3\hat{i} + 4\hat{j}$, $\vec{a} = 0.4\hat{i} + 0.3\hat{j}$, $t = 10$s

From 1st equatoin of motion.

$$a = \frac{v - u}{t}$$

$\therefore v = at\ tu$

$$\Rightarrow v = (0.4\hat{i} + 0.3\hat{j}) \times 10 + (3\hat{i} + 4\hat{j})$$

$$\Rightarrow 4\hat{i} + 3\hat{j} + 3\hat{j} + 4\hat{j}$$

$$\Rightarrow v = 7\hat{i} + 7\hat{j}$$

$$\Rightarrow |\vec{v}| = \sqrt{7^2 + 7^2} = 7\sqrt{2} \text{ unit.}$$

20. (b) Coordinates of moving particle at time '*t*' are $x = \alpha t^3$ and $y = \beta t^3$

$$v_x = \frac{dx}{dt} = 3\alpha t^2 \text{ and } v_y = \frac{dy}{dt} = 3\beta t^2$$

$$\therefore v = \sqrt{v_x^2 + v_y^2} = \sqrt{9\alpha^2 t^4 + 9\beta^2 t^4}$$

$$= 3t^2\sqrt{\alpha^2 + \beta^2}$$

21. (a) Using principal of conservation of linear momentum for horizontal motion, we have

$2mv_x = mu + mu \cos 60°$

$$v_x = \frac{3u}{4}$$

For vertical motion

$$h = 0 + \frac{1}{2}gT^2 \Rightarrow T = \sqrt{\frac{2h}{g}}$$

Let *R* is the horizontal distance travelled by the body.

$$R = v_x T + \frac{1}{2}(0)(T)^2 \text{ (For horizontal motion)}$$

$$R = v_x T = \frac{3u}{4} \times \sqrt{\frac{2h}{g}}$$

$$\Rightarrow R = \frac{3\sqrt{3}u^2}{8g}$$

22. (c) Given, $y = 2x - 9x^2$

On comparing with,

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta},$$

We have,

$$\tan\theta = 2 \text{ or } \cos\theta = \frac{1}{\sqrt{5}}$$

$$\text{and } \frac{g}{2u^2\cos^2\theta} = 9 \text{ or } \frac{10}{2u^2(1/\sqrt{5})^2} = 9$$

$\therefore$ $u = 5/3$ m/s

23. (d) *R* will be same for θ and $90° - \theta$.

Time of flights:

$$t_1 = \frac{2u\sin\theta}{g} \text{ and}$$

$$t_2 = \frac{2u\sin(90° - \theta)}{g} = \frac{2u\cos\theta}{g}$$

$$\text{Now, } t_1 t_2 = \left(\frac{2u\sin\theta}{g}\right)\left(\frac{2u\cos\theta}{g}\right)$$

$$= \frac{2}{g}\left(\frac{u^2\sin 2\theta}{g}\right) = \frac{2R}{g}$$

24. (b) For same range, the angle of projections are : θ and $90° - \theta$. So,

$$h_1 = \frac{u^2\sin^2\theta}{2g} \text{ and}$$

$$h_2 = \frac{u^2\sin^2(90° - \theta)}{2g} = \frac{u^2\cos^2\theta}{2g}$$

$$\text{Also, } R = \frac{u^2\sin 2\theta}{g}$$

$$h_1 h_2 = \frac{u^2\sin^2\theta}{2g} \times \frac{u^2\cos^2\theta}{2g}$$

$$= \frac{u^2}{16}\frac{u^2(2\sin\theta\cos\theta)^2}{g^2}$$

$$= \frac{R^2}{16}$$

or $R^2 = 16\, h_1 h_2$

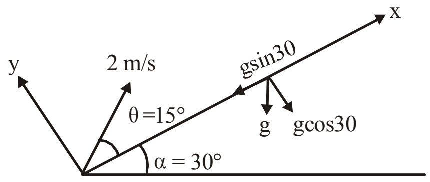
25. (a) On an inclined plane, time of flight (T) is given by

$$T = \frac{2u\sin\theta}{g\cos\alpha}$$

Substituting the values, we get

$$T = \frac{(2)(2\sin 15°)}{g\cos 30°} = \frac{4\sin 15°}{10\cos 30°}$$

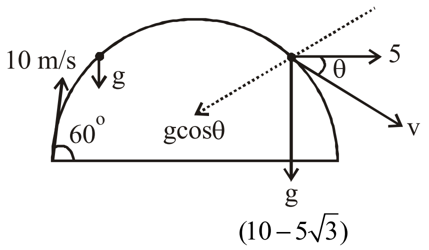
$$\text{Distance, S} = (2\cos 15°)T - \frac{1}{2}g\sin 30°(T)^2$$



$$= (2\cos 15°)\frac{4}{10}\frac{\sin 15°}{10\cos 30°} - \left(\frac{1}{2}\times 10\sin 30°\right)\frac{16\sin^2 15°}{100\cos^2 30°}$$

$$= \frac{16\sqrt{3}-16}{60} \simeq 0.1952\text{m} \simeq 20\text{cm}$$

26. **(b)**



Horizontal component of velocity

$v_x = 10\cos 60° = 5$ m/s

vertical component of velocity

$v_y = 10\cos 30° = 5\sqrt{3}$ m/s

After t = 1 sec.

Horizontal component of velocity $v_x = 5$ m/s

Vertical component of velocity

$v_y = |(5\sqrt{3}-10)|\ \text{m/s} = 10-5\sqrt{3}$

Centripetal, acceleration $a_n = \frac{v^2}{R}$

$$\Rightarrow R = \frac{v_x^2+v_y^2}{a_n} = \frac{25+100+75-100\sqrt{3}}{10\cos\theta} \quad ...(i)$$

From figure (using (i))

$$\tan\theta = \frac{10-5\sqrt{3}}{5} = 2-\sqrt{3} \Rightarrow \theta = 15°$$

$$R = \frac{100(2-\sqrt{3})}{10\cos 15} = 2.8\text{m}$$

27. **(a)** As we know, range $R = \frac{u^2\sin 2\theta}{g}$

and, area $A = \pi R^2$

$\therefore\ A \propto R^2$ or, $A \propto u^4$

$$\therefore\ \frac{A_1}{A_2} = \frac{u_1^4}{u_2^4} = \left[\frac{1}{2}\right]^4 = \frac{1}{16}$$

28. **(c)** Let 't' be the time taken by the bullet to hit the target.

$\therefore\ 700\text{ m} = 630\text{ ms}^{-1}\,t$

$$\Rightarrow\ t = \frac{700\text{m}}{630\text{ms}^{-1}} = \frac{10}{9}\text{sec}$$

For vertical motion,

Here, $u = 0$

$$\therefore\ h = \frac{1}{2}gt^2$$

$$= \frac{1}{2}\times 10\times\left(\frac{10}{9}\right)^2$$

$$= \frac{500}{81}\text{m} = 6.1\text{ m}$$

Therefore, the rifle must be aimed 6.1 m above the centre of the target to hit the target.

29. **(c)** From question,

Horizontal velocity (initial),

$$u_x = \frac{40}{2} = 20\text{m/s}$$

Vertical velocity (initial), $50 = u_y t + \frac{1}{2}gt^2$

$$\Rightarrow\ u_y\times 2 + \frac{1}{2}(-10)\times 4$$

or, $50 = 2u_y - 20$

or, $u_y = \frac{70}{2} = 35\text{m/s}$

$$\therefore\ \tan\theta = \frac{u_y}{u_x} = \frac{35}{20} = \frac{7}{4}$$

$$\Rightarrow\ \text{Angle } \theta = \tan^{-1}\frac{7}{4}$$

30. **(b)** From equation, $\vec{v} = \hat{i} + 2\hat{j}$

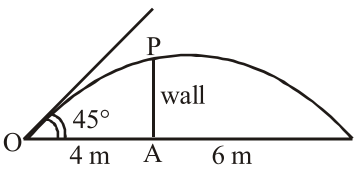
$\Rightarrow x = t$ ...(i)

$y = 2t - \frac{1}{2}(10t^2)$ ...(ii)

From (i) and (ii), $y = 2x - 5x^2$

31. **(b)**

32. **(b)**



As ball is projected at an angle 45° to the horizontal therefore Range = 4H

or $10 = 4H \Rightarrow H = \frac{10}{4} = 2.5\text{m}$

($\because$ Range = 4 m + 6 m = 10m)

Maximum height, $H = \frac{u^2\sin^2\theta}{2g}$

$$\therefore\ u^2 = \frac{H\times 2g}{\sin^2\theta} = \frac{2.5\times 2\times 10}{\left(\frac{1}{\sqrt{2}}\right)^2} = 100$$

or, $u = \sqrt{100} = 10\text{ ms}^{-1}$

Height of wall PA

$$= \text{OA}\tan\theta - \frac{1}{2}\frac{g(\text{OA})^2}{u^2\cos^2\theta}$$

$$= 4 - \frac{1}{2}\times\frac{10\times 16}{10\times 10\times\frac{1}{\sqrt{2}}\times\frac{1}{\sqrt{2}}} = 2.4\text{m}$$

33. **(d)** $R=\frac{u^2\sin^2\theta}{g}$, $H=\frac{u^2\sin^2\theta}{2g}$

H_{max} at $2\theta = 90°$

$$H_{max}=\frac{u^2}{2g}$$

$$\frac{u^2}{2g}=10 \Rightarrow u^2=10g\times 2$$

$$R=\frac{u^2\sin 2\theta}{g} \Rightarrow R_{max}=\frac{u^2}{g}$$

$$R_{max}=\frac{10\times g\times 2}{g}=20\,\text{metre}$$

34. **(a)** Let, total area around fountain

$A=\pi R_{max}^2$ ...(i)

Where $R_{max}=\frac{v^2\sin 2\theta}{g}=\frac{v^2\sin 90°}{g}=\frac{v^2}{g}$ ...(ii)

From equation (i) and (ii)

$$A=\pi\frac{v^4}{g^2}$$

35. **(a)** A projectile have same range for two angle
Let one angle be θ, then other is 90° – θ

$$T_1=\frac{2u\sin\theta}{g}, T_2=\frac{2u\cos\theta}{g}$$

then, $T_1T_2=\frac{4u^2\sin\theta\cos\theta}{g}=2R$

$$(\because R=\frac{u^2\sin^2\theta}{g})$$

Thus, it is proportional to R. (Range)

36. **(c)** Yes, Man will catch the ball, if the horizontal component of velocity becomes equal to the constant speed of man.

$$\frac{v_o}{2}=v_o\cos\theta$$

or $\theta=60°$

37. **(d)** Horizontal range is required

$$R=\frac{u^2\sin 2\theta}{g}=\frac{(10)^2\sin(2\times 30°)}{10}=5\sqrt{3}=8.66\,\text{m}$$

38. **(a)** Here, $R = 0.1$ m

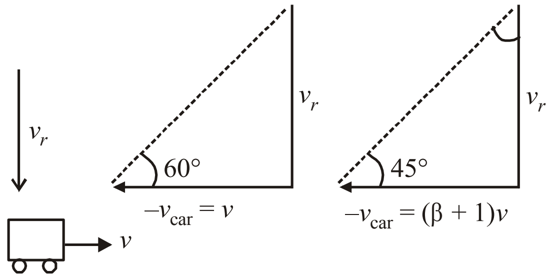
$$\omega=\frac{2\pi}{T}=\frac{2\pi}{60}=0.105\ \text{rad/s}$$

Acceleration of the tip of the clock second's hand,

$$a=\omega^2R=(0.105)^2(0.1)=0.0011=1.1\times 10^{-3}\ \text{m/s}^2$$

Hence, average acceleration is of the order of 10^{-3}.

39. **(d)** The given situation is shown in the diagram. Here v_r be the velocity of rain drop.



When car is moving with speed v,

$\tan 60°=\frac{v_r}{v}$ ...(i)

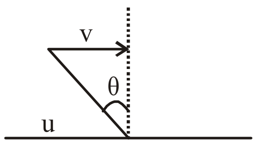
When car is moving with speed $(1+\beta)v$,

$\tan 45°=\frac{v_r}{(\beta+1)v}$ ...(ii)

Dividing (i) by (ii) we get,

$$\sqrt{3}v=(\beta+1)v \Rightarrow \beta=\sqrt{3}-1=0.732.$$

40. **(c)** $\sin\theta=\frac{u}{v}=\frac{2}{4}=\frac{1}{2}$

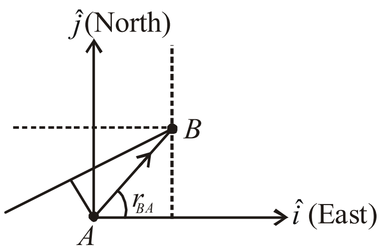


or $\theta = 30°$

with respect to flow,

$= 90° + 30° = 120°$

41. **(b)**



$$\vec{v}_A=30\hat{i}+50\hat{j}\ \text{km/hr}$$

$$\vec{v}_B=(-10\hat{i})\ \text{km/hr}$$

$$r_{BA}=(80\hat{i}+150\hat{j})\ \text{km}$$

$$\vec{v}_{BA}=\vec{v}_B-\vec{v}_A=-10\hat{i}-30\hat{i}-50\hat{i}=40\hat{i}-50\hat{j}$$

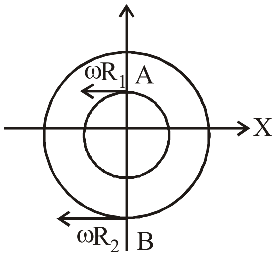
$$t_{minimum}=\frac{|(\vec{r}_{BA})\cdot(\vec{v}_{BA})|}{|(\vec{v}_{BA})|^2}$$

$$=\frac{|(80\hat{i}+150\hat{j})(-40\hat{i}-50\hat{j})|}{(10\sqrt{41})^2}$$

$$\therefore\ t=\frac{10700}{10\sqrt{41}\times 10\sqrt{41}}=\frac{107}{41}=2.6\ \text{hrs.}$$

42. **(c)** From, $\theta = \omega t = \omega \frac{\pi}{2\omega} = \frac{\pi}{2}$

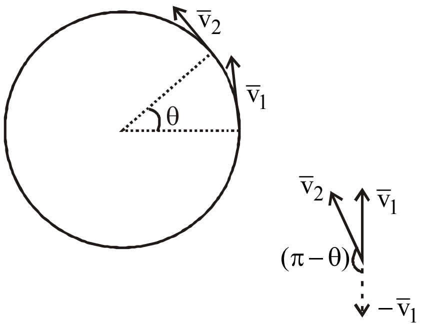
So, both have completed quater circle



Relative velocity,

$v_A - v_B = \omega R_1(-\hat{i}) - \omega R_2(-i) = \omega(R_2 - R_1)i$

43. **(d)**



Change in velocity,

$|\Delta\vec{v}| = \sqrt{v_1^2 + v_2^2 + 2v_1 v_2 \cos(\pi - \theta)}$

$= 2v\sin\frac{\theta}{2} \qquad (\because |\vec{v}_1| = |\vec{v}_2|) = v$

$= (2 \times 10) \times \sin(30°) = 2 \times 10 \times \frac{1}{2}$

$= 10$ m/s

44. **(c)** Speed, V = constant (from question)

Centripetal acceleration,

$a = \frac{V^2}{r}$

ra = constant

Hence graph (c) correctly describes relation between acceleration and radius.